Published in:
Automation and Remote Control, RF,
Vol. 62, No. 5, 2001, pp. 94–104.

An Improved Reduction Algorithm to Check Hypotheses for the Multicollinear Regression Model

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It is proposed a method to check hypotheses about values of the variance and mean value of the linear function of the state vector by an ensemble of regression experiments in the multicollinearity conditions. The method is suggested to solve high-dimensionality problems arising upon testing complex technical systems.

1. Introduction and problem formulation

The main propositions of this paper are based on results of article [1], which describes a method for checking hypotheses about values of the variance and mean value of the goal characteristic of a static linear system by an ensemble of regression experiments. The method was successfully tested for small-dimensionality problems, but attempts to apply it to the performance control with a high-dimensionality vector ($m > 20$) wasn't successful. For high-dimensionality problems there was need of some modifications and additions to the algorithm [1]. Necessary modifications are discussed below, and the results of testing the improved algorithm are presented.

Let the observation equation in an ensemble of N regression experiments be as follows:

$$y = Hx + \eta,$$

where y is the observation vector of dimensionality n; x is the state vector of dimensionality m, $m \le n$; η is the noise vector with normal distribution and certain variance matrix $D[\eta] = B > 0$; H is the full-rank design matrix. We assume that the state vector has normal distribution with independent components, that is, $\mathrm{D}[x] = \mathrm{diag}$. Next boundary values of parameters are given:

$$\left|M[x_i]\right| \le M_{\max,i}\,, \tag{1}$$

$$0 < D_{\min,i} \le \mathrm{D}[x_i] \le D_{\max,i}\,,\ i = \overline{1,m}\,. \tag{2}$$

The goal characteristic is defined by the m_1 first state components as

$$L = \sum_{i=1}^{m_1} x_i\,. \tag{3}$$

In the problem of variance checking, we have to construct, using a sample of vectors y of size N, a test for checking the zero hypothesis

$$\mathrm{D}[L] = \sum_{i=1}^{m_1} \mathrm{D}[x_i] = D_{\mathrm{TTT}} \tag{4}$$

against the alternative

$$\sum_{i=1}^{m_1} \mathrm{D}[x_i] = D_A > D_{\mathrm{TTT}}. \tag{5}$$

The problem of checking the mean value has a similar formulation; the zero and alternative hypotheses are represented by expressions like (4) and (5).

Regressors of the design matrix are known to be able to form groups. We introduce the family $G\bullet = \{G_j,\ j = \overline{1,q}\}$ of groups of numbers such that

$$G_1 \cap G_j = \varnothing \ \text{ if } \ i \neq j; \ \bigcup_{j=1}^{q} G_j = \{1, \ldots, m\}; \ G_j \neq \varnothing,$$

$$G_j \subset \{1, \ldots, m_1\} \ \text{ or } \ G_j \subset \{m_1 + 1, \ldots, m\}, \ \ j = \overline{1,q},$$

introduce a q-dimensional vector I whose components are selected from the set {1, 2, ... , m} and ordered so that for some natural r

$$I_j \leq m_1 \ \text{ if } \ j \leq r \ \text{ or } \ I_j > m_1 \ \text{ if } \ r < j \leq q, \tag{6}$$

and denote by $\mathcal{E}_B^n$ the Euclidean space with the scalar product $(a,\ b)_B = a^{\mathrm{T}} B^{-1} b$. Let $\|a\|_B \equiv (a,a)_B^{1/2}$. The subscript i denotes the i-th column-vector of the matrix.

We assume that there exist the vector I and the family $G\bullet$ such that in $\mathcal{E}_B^n$ the vectors

$$h_{*i} = H_{*i} - H_{*I_j}, \ i = \overline{1,m}, \ j \ \text{ such that } \ i \in G_j, \tag{7}$$

are small in norm. In other words, for an appropriate choice of I and $G\bullet$, the reduced matrix

$$\overset{\mathrm{o}}{H} = (H_{*I_1}, H_{*I_2}, \ldots, H_{*I_q}) \tag{8}$$

defines the regressors' "grouping centers", so that regressors H_{*i} for $i \in G_j$ are grouping in the neighborhood of the j-th center H_{*I_j}.

2. Essence of the method of reduction

Since the methods for checking the variance and mean value of the goal characteristic are not essentially different, we confine our consideration to the method of checking the hypotheses about the values of variance D[L]. The main result of [1] consists in generating the permissible set of estimates $\hat{d}_I$ of the useful parameter (variance) which define the corresponding set $\{\pi_{I,v}\}$ of tests with the critical domain $\hat{d}_I > v$, where the power-optimal test is sought. The estimates $\hat{d}_I$ are defined by

every possible reduced design matrices $\overset{o}{H}$; are constructed using the reduced statistics

$$\hat{z} = \overset{o}{F}^{-1} \overset{o}{H}^{T} B^{-1} y; \quad \overset{o}{F} = \overset{o}{H}^{T} B^{-1} \overset{o}{H}; \tag{9}$$

and have the analytical form

$$\hat{d}'_I = \sum_{j=1}^{r} S_j; \quad S_j = \frac{1}{N-1} \sum_{k=1}^{N} \left(\hat{z}_{jk} - \bar{\hat{z}}_j \right)^2, \tag{10}$$

where $\hat{z}_{jk}$ is the *k*-th sampled value of the *j*-th component of the vector $\hat{z}$. Since the estimates $\hat{d}_I$ were introduced as the result of computer calculations, they differ from $\hat{d}'_I$ owing to calculation errors.

To calculate the power criterion of test $\pi_{I,v}$, the following guaranteed characteristics of the estimate $\hat{d}_I$ are given:

$$\begin{aligned} M^0_{\max}(I) &= \max_{1\text{-}2,4} \mathrm{M}[\hat{d}_I]; \quad M^A_{\min}(I) = \min_{1\text{-}2,5} \mathrm{M}[\hat{d}_I]; \\ D^0_{\max}(I) &= \max_{1\text{-}2,4} \mathrm{D}[\hat{d}_I]; \quad D^A_{\max}(I) = \max_{1\text{-}2,5} \mathrm{D}[\hat{d}_I]; \end{aligned} \tag{11}$$

where indices under "max" and "min" point to the numbers of the conditions for which the extremum is sought. For the given level of significance $\alpha \le 1/2$, expressions for the guaranteed strength of test $\pi_{I,v}$ and strength-optimal tolerance boundary $v(I)$ are expressed as follows:

$$\begin{aligned} \underline{\beta}_\alpha(I) &\equiv \Phi\left(\frac{M^A_{\min}(I) - v(I)}{\sqrt{D^A_{\max}(I)}} \right); \\ v(I) &\equiv M^0_{\max}(I) - \Phi^{-1}(\alpha) \sqrt{D^0_{\max}(I)}, \end{aligned} \tag{12}$$

where Φ is the function of standard normal distribution and Φ^{-1} is its inverse function. As was indicated in [1], function (12) has the sense of guaranteed power only if $\underline{\beta}_\alpha(I) \ge 1/2$, that must be taken into account when applying the local optimization algorithms (see below).

According to [1, 2], *by the reduction of the multicollinear observation model we name the procedure to solve the statistical problem using the reduced statistics (9) and stepwise determination of the reduced design matrix* $\overset{o}{H}$*, which is optimal in some criterion.* We shell consider the method of reduction as means to solve fairly different problems [1, 2].

Let us introduce a *q*-dimensional vector $\overset{o}{x}$ of group components of the system state vector:

$$\overset{o}{x}_j = \sum_{i \in G_j} x_i , \quad j = \overline{1,q} . \tag{13}$$

The following theorem establishes the possibility of estimating the vector of group components using the reduced statistics.

Theorem. Convergence in probability $\hat{z} \xrightarrow{P} \overset{o}{x}$ takes place for

$$\|h_{*i}\|_B \to 0 , \quad i = \overline{1,m} \text{ and } \|B\|_E \to 0 . \tag{14}$$

Proof. Denote by $\overset{o}{\mathcal{H}}$ the linear vector shell constructed on the regressors of the matrix $\overset{o}{H}$. Represent vectors (7) as the orthogonal resolutions in $\overset{o}{\mathcal{E}}{}^n_B$:

$$h_{*i} = h^{\|}_{*i} + h^{\perp}_{*i} \text{ where } h^{\|}_{*i} \in \overset{o}{\mathcal{H}} , \ (h^{\perp}_{*i}, a)_B = 0 \text{ for any } a \in \overset{o}{\mathcal{H}} .$$

Carrying out development as in [1, sec. 2], we readily obtain:

$$\Delta \hat{z} \equiv \hat{z} - \overset{o}{x} = Rx + \hat{\delta}_\eta , \tag{15}$$

where

$$R = \left(\overset{o}{H}{}^{\mathrm{T}} \overset{o}{H} \right)^{-1} \overset{o}{H}{}^{\mathrm{T}} \left(h^{\|}_{*1}, \ldots, h^{\|}_{*m} \right); \tag{16}$$

$$\hat{\delta}_\eta = \overset{o}{F}{}^{-1} \overset{o}{H}{}^{\mathrm{T}} B^{-1} \eta .$$

Since $\|h_{*i}\|_B \lambda^{1/2}_{\max}(B) \geq \|h^{\|}_{*i}\|_B \lambda^{1/2}_{\max}(B) \geq \|h^{\|}_{*i}\|_E$, where $\lambda_{\max}(\cdot)$ is the maximal eigenvalue of the matrix, convergence $R \to 0$ exists under conditions (14). Since $\mathrm{D}[\hat{\delta}_\eta] = \overset{o}{F}{}^{-1} \to 0$ and $\mathrm{M}[\hat{\delta}_\eta] = 0$ under the same conditions, we establish validity of the theorem.

To interpret (15), we could note that the error of estimating the vector of group components depends on two factors: the error $\hat{\delta}_\eta$ of the LSM-estimate obtained by the reduced model of the form $y \approx \overset{o}{H} \overset{o}{x} + \eta$ and the error Rx caused by inadequacy of the model. The latter error depends only on the components x_i eliminated from the reduced model. Indeed, $h_{*i} = 0$ for $i \in I$, and consequently,

$$Rx = \sum_{i \notin I} R_{*i} x_i . \tag{17}$$

Therefore, *when generating the reduced design matrix by selecting the regressor grouping centers in sufficiently low noise, the group components of state can be estimated by reduced statistics.* This affirmation warrants the method of reduction as the mean for solving a wide class of statistical problems [3]. In the problems of checking the variance D[L], for example, estimate (10) can be obtained from

$$\mathrm{D}[L] = \sum_{j=1}^{r} \mathrm{D}[\overset{\circ}{x}_j]$$

where the true variances in the right-hand side are replaced by close sampled variances of $\xi_j \approx \overset{\circ}{x}_j$. The problem of estimating the goal characteristic (3) by a single experiment under multicollinearity was solved in [2] by reducing the model. The estimate proposed in [2] is obtained from the above statement by summing the components ξ_j, $j = \overline{1,r}$. Additionally, closeness of the statistics ξ to the vector $\overset{\circ}{x}$ was used in the optimality studies of the solutions obtained in [1, 2].

3. Comparison of the stepwise schemes of regressor selection

The regressor selection algorithm used in the present paper to solve the problem of checking the variance D[L] was elaborated and successfully approved for the problem of estimating the goal characteristic L by a single experiment [2]. Its application to optimizing the test $\pi_{l,v(l)}$ by the power criterion (12) cannot be stated successful because solution was possible only for small dimensionality ($m \approx 5$). To make the computational scheme applicable to larger dimensionalities of the problem, we shall analyze the existing stepwise schemes of regressor selection [4]. Consideration is given to the class of problems where the criterion function is the generalized measure of the error of reduced estimation $\Delta\xi$.

Algorithm of successive elimination generates the reduced design matrix by stepwise elimination of regressors from the original matrix. To make it reliable, one needs reliable calculations of the selection criterion for a large number q of regressors retained in $\overset{\circ}{H}$. The optimization criterion of problem (12) does not meet this requirement: attempts to calculate this criterion for $q \approx m$ result in a abend of the program because of attempts to divide by zero upon matrix inversion. Nevertheless, the scheme proved operable for a similar problem using the regressor grouping criteria of the cluster analysis [5].

In the algorithm of successive attachment the dimensionality of the reduced design matrix successively increases beginning from $q = 1$. At the first step, a single regressor making up the matrix $\overset{\circ}{H}$ is chosen in the optimal manner. According to (15)—(17), at the first step the chosen regressor is averaged over the entire orbit such that the majority of the vectors h_{*i} is small in norm. This regressor has, if possible, the number of the dominating component x_i to decrease the estimation error according to (17). Figure 1 depicts an example of grouping the regressors of the original design matrix (projected on the plane). If the components x_i are sufficiently small for $i \in G_3$, then $\overset{\circ}{H}_{*1}$, which is the "intermediate" regressor between the groups G_1 and G_2, is the anticipated result of the first step of the scheme. Since the estimation error $\Delta\xi$ is affected substantially by the components x_i eliminated from the reduced model, at the

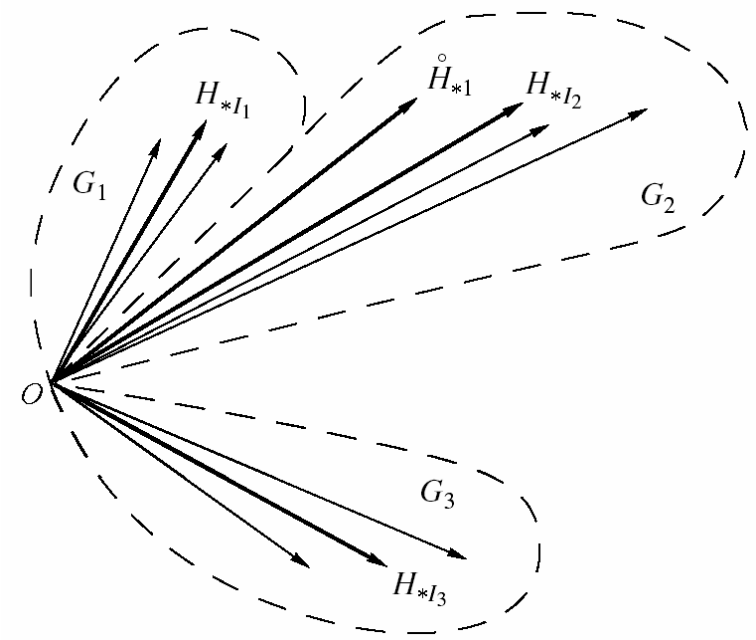


Fig.1. Example of regressor grouping: G_j — groups of close regressors; H_{*I_j} — centers of grouping; $\overset{o}{H}_{*1}$ — first attached regressor

first steps of the algorithm (when $q = 1, 2$) the errors $\Delta\hat{\varepsilon}_j$ have to be comparatively great for a large dimensionality ($m > 20$) of the problem. So, at the beginning of the run, one can get $\beta_\alpha < 1/2$, that is, criterion (12) is no more the generalized measure of the estimation error $\Delta\hat{\varepsilon}$ and its local optimal value is far from the optimal value. At the same time, in testing problems of small-dimensionality ($m \approx 5$), even the first step of the attachment algorithm can provide a satisfactory result, if only $\beta_\alpha \geq 1/2$ is attained.

Algorithm of attachment-elimination needs the initial reduced matrix $\overset{o}{H} = \overset{o}{H}_{\text{init}}$ from which the stepwise optimization is under way by attempts (i) to attach a regressor to the matrix $\overset{o}{H}$; (ii) eliminate a regressor from the matrix $\overset{o}{H}$; and (iii) replace pairwise the attached regressor by the eliminated one. This scheme gains an advantage over the last algorithm in possible replacing of unsuccessfully chosen regressors in $\overset{o}{H}$, but for great m it is substantially slower.

4. Combined stepwise algorithm and the possibility to improve it

It was proposed in [1, 2] to construct the local optimal matrix $\overset{o}{H}$ using the combined stepwise algorithm of successive attachment with an attempt of pairwise replacement of the attached regressors after the current attachment step. This scheme permits to correct the consequences of unsuccessful initial attachment steps, but

because of the nonphysical nature of criterion (12) the possibility of unsatisfactory solution remains. Further we shall describe the algorithm in formal terms and ground its improvements for high-dimensionality problems of checking the variance D[L]. To describe the algorithm, we consider the set $\{I\}$ of all possible integer vectors introduced above by inequalities (6) for the given characteristics of dimensionalities m_1 and m.

Definition 1. By the neighborhood of the vector I^0 on the subset $\mathcal{G}$ of the feasible set $\{I\}$ is named the set of vectors $I \in \mathcal{G}$ such that the sets of values of the components of the vectors I and I^0 differ at most in one element.

If the subset $\mathcal{G} \subset \{I\}$ consists of all feasible vectors of the given dimensionality q, then the corresponding neighborhood will be denoted by $O^q(I^0)$. Here it is possible that dim $I^0 \neq q$ and, correspondingly, $I^0 \notin O^q(I^0)$.

Definition 2. By the extended neighborhood of the vector $I \in \{I\}$ is named the set $\overline{O}(I) = O^k(I) \cup O^{k+1}(I)$, k = dim I.

The next algorithm is valid to establish the local optimal solution of the problem of minimization of some criterion function $Q(I)$ over an extended neighborhood $\overline{O}(I)$. To solve the problem under consideration, one must assume that $Q = -\underline{\beta}_\alpha(I)$.

Algorithm 1

1. Step 1:

1.1. The one-dimensional vector $I^1 = i_1$ such that $Q(i_1) = \min Q(i)$ in $i = \overline{1,m}$ is determined by overselection the numbers $i = \overline{1,m}$.

1.2. If $m = 1$ then go to step 4.

2. Set $q = 1$.

3. $(q + 1)$-th step:

3.1. The vector I such that $Q(I) = \min Q(I)$ in $I \in O^{q+1}(I^q)$ is determined by total overselection in the neighborhood $O^{q+1}(I^q)$.

3.2. The $(q + 1)$-dimensional vector I^2, giving local minimum over the neighborhood $O^{q+1}(x)$, is determined for the initial vector of search $I^0_{(q+1)} = I$ (see Algorithm 2).

3.3. If $Q(I^q) < Q(I^2)$ then go to step 4.

3.4. Set $I^{q+1} = I^2$ and then set $q = q + 1$.

3.5. If $q = m$ then go to step 4, otherwise, go to step 3.

4. End: the local minimum of the criterion function over the extended neighborhood $\overline{O}(x)$ is determined at the point I^q.

In this algorithm, step 3.2 consists in local optimization on the set of vectors I of the given dimensionality dim $I = q + 1$. This problem is solved by the following procedure.

Algorithm 2

1. Set $i = 0$.
2. $(i + 1)$-th step:

2.1. Checking the point $I^i_{(q+1)}$ for local minimum. By overselection the vectors of the neighborhood $O^{q+1}(I^i_{(q+1)})$ it is sought any vector I^2 for which $Q(I^2) < Q(I^i_{(q+1)})$. If there is no such point, go to step 3.

2.2. Set $I^{i+1}_{(q+1)} = I^2$ and then $i = i + 1$.

2.3. Go to step 2.

3. Set $I^2 = I^i_{(q+1)}$. End: the local minimum on the set of $(q + 1)$-dimensional vectors I over the neighborhood $O^{q+1}(\chi)$ is determined at the point I^2.

Let us consider the necessary modifications and improvements of the above algorithm for solving the high-dimensionality problems of checking the variance of the goal characteristic.

First modification of the algorithm concerns of the power criterion (12). Let's write out the problem of its maximization in the equivalent form:

$$\frac{M^A_{\min}(I) - M^0_{\max}(I) - \left|\Phi^{-1}(\alpha)\right|\sqrt{D^0_{\max}(I)}}{\sqrt{D^A_{\max}(I)}} \to \max_I \qquad (18)$$

It proves to be that the stepwise maximization of the last criterion by Algorithm 1 can result in an unacceptable solution in the presence of a satisfactory solution (high-power test).

On the normal execution of the reduction algorithm, in the course of regressor attachment the reduced statistics (9) estimates more and more group components of the state vector (13) and the errors of estimation decrease with each step so that the error of estimation of variance (10) decreases as well. Let some neighborhood of the vector I be considered at an early stage of attachment, when the accuracy of the estimate $\hat{d}'$ is low. Let it be established that in this neighborhood the numerator in (18) is negative for all estimates of the variance. Then it would have been chosen a solution with the maximal variance $D^A_{\max}(I)$, for which (18) is most close to zero (maximal), i.e. the matrix $\overset{o}{H}$ retains its multicollinearity! Additionally, it happened once in trial calculations that from the possible estimates $\hat{d}_I$ a close-to-zero estimate was chosen for which the numerator in (18) was sufficiently small. Anyhow, at the beginning of optimization an unsuccessful solution was chosen, thus aggravating the final result.

The negative numerator in (18) can be resulted of low accuracy of the estimates $\hat{d}_I$ if $M^A_{\min} < M^0_{\max}$ for all vectors I of the neighborhood, but in the presence of estimates of high accuracy, the numerator will be inevitably negative for

a sufficiently low level of significance α (because $|\Phi^{-1}(\alpha)| \to +\infty$ for $\alpha \to +0$). If in the neighborhood under study there was at least one solution with a positive criterion (18), then it would be chosen and process continued by the normal selection of regressors, but usually the initial step of optimization (where $q = 1, 2$) has no such solutions. The cause of this "computational catastrophe" is that the function $\underline{\beta}_\alpha(I)$ in (12) is the test power only if $\underline{\beta}_\alpha(I) \geq 1/2$, i.e. if the numerator in (18) is nonnegative. We propose the following corrected minimization criterion guaranteeing normal selection of the regressors for large-scale models:

$$Q(I) = \begin{cases} \max\left(\left|M^A_{\min} - D_A\right|, \left|M^0_{\max} - D_{\mathrm{TTT}}\right|\right) & \text{if } M^A_{\min} < M^0_{\max}; \\ -\underline{\beta}_\alpha(I) & \text{if } \underline{\beta}_\alpha(I) \geq 1/2; \\ -0.5\Phi\left(\dfrac{M^A_{\min} - M^0_{\max}}{\sqrt{D^A_{\max}}}\right) & \text{if } \underline{\beta}_\alpha(I) < 1/2 \text{ and } M^A_{\min} \geq M^0_{\max}, \end{cases} \tag{19}$$

where the guaranteed characteristics (11) are parameterized by the integer vector I.
At the early stage of selection, when $M^A_{\min} < M^0_{\max}$ and the numerator in (18) is to be negative, selection by criterion (19) is intended for reducing the guaranteed shifts of the estimate $\hat{d}_I$. Next, one encounters a solution for which $M^A_{\min} \geq M^0_{\max}$. For the given $\alpha < 1/2$ it is still possible that $\underline{\beta}_\alpha < 1/2$, but

$$\underline{\beta}_{0.5}(I) = \Phi\left(\frac{M^A_{\min} - M^0_{\max}}{\sqrt{D^A_{\max}}}\right) \geq 1/2,$$

that is, the function $\underline{\beta}_{0.5}(I)$ acquires here the sense of the power of test of size 0.5. Criterion (19) enables one to "capture" the greatest-power solution $\underline{\beta}_\alpha(I) \geq 1/2$, if it exists, or to choose in the neighborhood the strongest solution for the significance level 0.5. In the latter case, run of the algorithm provides increase of the criterion $\underline{\beta}_{0.5}(I)$ owing to growth of difference $M^A_{\min} - M^0_{\max}$ and decrease of variance $D^A_{\max}$. In doing so, the variance $D^0_{\max}$ usually decreases (because $D^A_{\max} \geq D^0_{\max}$ to within computational errors) and the numerator in (18) increases on the whole. Under reasonable choice of α, a solution with $\underline{\beta}_\alpha(I) \geq 1/2$ will be determined, and optimization by the power criterion will be continued until the local optimum reached.

Second modification of reduction algorithm concerns of the possibility of improving the solution by continuing the attachment from the point of local minimum. Modification consists in replacing step 3.3 of Algorithm 1 by a null operation and checking at step 3.5 the condition for attainment of the predefined

number of attached regressors $q_{\max}$. In one of three trial calculations, compulsory attachment resulted in a more powerful test.

Third modification consists in increasing the rate of regressor selection by separating the subproblem of maximization of the variance $D[\hat{d}_I]$ as an individual step. This subproblem lies in maximization of the convex function on the feasible polyhedron [1], so, we have sufficiently laborious problem of the concave programming [6, 7]. In practice, however, it turned out that the approximate solution by the simplex method usually coincides with the precise solution. Therefore, it is reasonable to select regressors by calculating the variances $D^0_{\max}$, $D^A_{\max}$ by means of simplex technique and then verify the calculations of the maximization criterion only for the points I, optimal in each of the checked neighborhoods. The checked neighborhoods must be surveyed with more precise calculation of the variances only if the differences are detected in the results of "fast" and "refined" calculations of criterion (19). Otherwise, the obtained solution will be certainly locally optimal since for the remaining solutions of the neighborhood the refined variances can only reduce criterion (19).

5. Results of approval

As was noted in [1], the new method of checking the hypotheses about the goal characteristic can be used in flying testing of the ballistic missiles where the main flight characteristic, variance of the hitting error (grouping of fire), must be checked to satisfy the desired performance. To approve the new method, it was developed a test observation model describing launching of the ballistic missile with a standard inertial guidance system — three-axes forced gyrostabilizer with string accelerometers mounted on the platform [8]. It is planned to use the data of the tracking measurements done by the "Mistrem" phase-angle-meter system [9]. The hitting error (range deviation) is proposed only due to the instrument errors of the gyrostabilizer. The state vector of the non-normalized model [3] has the following components: $\overset{*}{x}_1 — \overset{*}{x}_2$ are the errors in the scale coefficients, respectively, of the x-accelerometers and y-accelerometers; $\overset{*}{x}_3 — \overset{*}{x}_5$ are angular rates of the constant platform drift; $\overset{*}{x}_6 — \overset{*}{x}_8$ is the imbalance of the gyro units; $\overset{*}{x}_9 — \overset{*}{x}_{11}$ are the errors of the initial gyro alignment; $\overset{*}{x}_{12} — \overset{*}{x}_{14}$ are the static stabilization errors; $\overset{*}{x}_{15} — \overset{*}{x}_{17}$ are the accelerometers' "nonzeros" and $\overset{*}{x}_{18} — \overset{*}{x}_{21}$ are the alignment errors of the antenna field of the tracking radar. These components must be appropriately normalized when composing the vector x.

The model includes the pretest variances $D_{apr}[x_i]$, $i = \overline{1,21}$, of the components of the state vector. The variances differ substantially (by the orders of magnitude); three variances (No. 8, 12, 13) are of dominating values. For the boundary values of the experiments, it was assumed that

$$D_{\min,i} = 1.25^{-2}\,\mathrm{D}_{\mathrm{apr}}[x_i] \quad i = \overline{1,21};$$

$$D_{\max,i} = \begin{cases} 1.5^2\,\mathrm{D}_{\mathrm{apr}}[x_i] & \text{if } i \neq 8,12,13; \\ 1.3^2\,\mathrm{D}_{\mathrm{apr}}[x_i] & \text{if } i = 8,12,13. \end{cases}$$

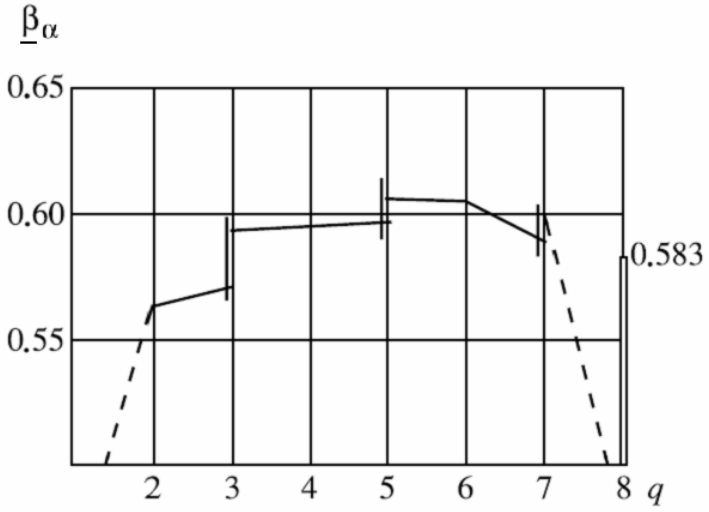
The acceptance and rejection values of variance were as follows:

$$D_{\mathrm{TTT}} = \sum_{i=1}^{17} \mathrm{D}_{\mathrm{apr}}[x_i];\ D_{\mathrm{TTT}} = \lambda^2 D_{\mathrm{TTT}},\ \lambda = 1.25.$$

A run of $N = 7$ firings was processed; the design matrix was formed from forty measurements of the velocity (correspondingly, $n = 120$). For the significance level of the required test, it was assumed that $\alpha = 0.25$.

Although this model is extremely simplified, it has the characteristic properties of the real model: the design matrix is multicollinear; the state vector is determined with great prior uncertainty[1]; the components of the state vector differ by orders of magnitude.

The results of applying the modified algorithm to this model are plotted in Fig. 2 as the test power vs. the number of attached regressors. The vertical lines refer to the cases where power was increased by replacing the attached regressor by a remote one (step 3.2 of Algorithm 1), the dashed lines refer to the cases where $\underline{\beta}_\alpha(I) < 1/2$. The most powerful test $\underline{\beta}_\alpha = 0.61$ was obtained for $q = 5$ attached regressors.



Let us compare the result presented with the results of the conventional method of checking by the sample of hits of the warhead. In the latter case, it is applicable the test of likelihood ratio, which consists in comparing the sampled variance of the error of hitting with the tolerance boundary [11]. For the significance level α, its power is as follows:

[1] Regularization of the information matrix by the parameters $D_{\max,i}$ for estimating the state vector [10] does not secure the desired estimation accuracy.

$$\beta = \Pr\left(\chi^2 > \chi^2_{1-\alpha,N-1} \big/ \lambda^2\right),$$

where the random quantity χ^2 has $N - 1$ degrees of freedom; $\chi^2_{x,f}$ is the quantile of the level x of the χ^2-distribution with f degrees of freedom. One can see that in the testing example $\beta = 0.54$, whereas the power $\beta = 0.61$ is attained if the sample volume is increased to $N = 11$.

Volumes of the considered sample $N = 7$—11 are characteristic of the flight tests [12], and as one can see, the probability of accepting an off-grade system is rather high (of the order of 40%). Although cases are known where the flight tests failed to identify low missile accuracy [12], under the given funding to increase the sample volume is inadvisable from the point of view of missile system efficiency [13]. This fact is in agreement with the findings of the reliability theory that the error of the first or second kind should be optimized by efficiency criterion and that substantial error level of performance control of complex systems is possible even after optimization [14].

6. Conclusions

The advantage of the new method of check of grouping of fire consists mostly in the possibility to check grouping in the required application conditions, rather than in higher test confidence. To check system properties in the required conditions it is sufficient to normalize the original model like it was recommended in [3], whereas by the sample of hitting errors grouping is estimated only for the test conditions.

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